### Lab 9 – Quaternions arithmetics

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# Learning objectives

## Exam objectives

By the end of this lab you should be able to (pen and paper):

* Perform quaternion arithmetics: addition, subtraction, multiplication, division and exponentiation
* Simplify quaternion (alfa)numerical expressions
* Convert between both the algebraic, the trigonometric and the vector representation of a quaternion

We advise you to **make your own summary of topics** which are new to you.

## Supportive objectives

Do not try to use geogebra, because it will not work. Use the **VecMath shell** instead, availed via LEHO/AMP2. This is verify all of your quaternion arithmetic.

# Exercises

## Basic exercises

The **quaternion conjugate of q** is here typeset as **q∗**,   
the **quaternion inverse of q** is here typeset as **q-1** and   
the **normalized quaternion of q** is here typeset as **qn**.

Given are the two quaternions   
 **and**

for the underneath exercises 1 to 6. For your convenience, do express both by their **vector representations** and which offers as well more insights as shortens the calculation load in general.

### Vector representation of quaternions

**Exercise 1**: Calculate pen and paper the quaternion

* Addition
* Subtraction
* The particular addition

**Exercise 2:** Firstly calculate pen and paper the quaternion using the vector represented multiplication formula:

* **Multiplication**
* And the **norm of this product**

For the multiplication you can use 2 methods.

1. By using the formula in
2. By using distributivity : (1 + 2i + 3j + 4k) . (2 + 3i + 4 j + 5k) and simplify

Try both methods! The results should be the same.

Then verify each by the application **VecMath shell**.

**Exercise 3**: Firstly **calculate pen and paper** the

* Quaternion norm
* Normalized quaternion
* Inverse quaternion

Then verify each by the application **VecMath shell**.

using **quaternion q = 2 + 3i + 4 j + 5 k** as input

**Exercise 4**: Firstly **calculate pen and paper** the

* Quaternion norm
* Normalized quaternion
* Inverse quaternion
* Then verify each by the application **VecMath shell**.
* using **quaternion p = 1 + 2i + 3 j + 4 k** as input

**Exercise 5**: Firstly verify pen and paper whether these quaternion multiplications satisfy

* *yes*
* *yes*
* Then verify each by the application **VecMath shell** using **quaternions p = 1 + 2i + 3 j + 4 k** and **q = 2 + 3i + 4 j + 5 k** as input.

**Exercise 6**: Firstly verify pen and paper whether the

* Quaternion inverse of the product
* the product of the inverses in **reverse** order known as the so-called *‘Socks and Boots’* rule (familiar in *matrix algebra*)
* Then verify each by the application **VecMath shell** using **quaternions p = 1 + 2i + 3 j + 4 k** and **q = 2 + 3i + 4 j + 5 k** as input.

## Bridging exercises

### Trigonometric representation of quaternions

Given is the general **quaternion**  in its **trigonometric**

**representation** with

**Exercise 7**: Upon the above verifications, now **prove** by

pen and paper the identity

**Exercise 8**: Upon the above verifications, now **prove** by

and by the quaternion exponentiation pen and

paper, the identity

# References

## Basics

### English maths dictionary

<http://www.mathwords.com>

### Wolframalpha

<https://www.wolframalpha.com/input/?i=quaternions>

## Demos in art and programming

### Quaternion based camera

<https://www.gamedev.net/articles/programming/math-and-physics/a-simple-quaternion-based-camera-r1997/>